International Journal of Business Management and Economic Review
Vol. 1, No. 04; 2018
ISSN: 2581-4664

AN APPLICATION OF BOOTSTRAP AND JACKKNIFE TECHNIQUES TO AUTOMOTIVE INDUSTRY

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ABSTRACT
The aim of this study is to apply repeated sampling methods which are used frequently in recent years on the automotive industry. Resampling methods are based on the distribution of B repeated samples, each of which is drawn with replacement from the observed data set. It is aimed to reduce the standard error of the variable of interest. In this study engine power, weight and fuel consumption are the variables of interest and the estimations of the variables obtained with the help of the generated empirical distribution. Bootstrap-t, percentage and bias corrected and accelerated confidence intervals were obtained for 1.6 and 1.8 injection vehicles with B=1000 repeated samples and the results were evaluated statistically. The results were obtained using the R software program and the evaluation was done with confidence intervals and graphs.

Keyword: Bootstrap, confidence intervals, jackknife, resampling.

INTRODUCTION
Resampling is a collection of sample-based methods used to demonstrate statistical significance or to determine the upper and lower bounds of a confidence interval when parametric statistical methods cannot be applied. These methods are an alternative to classical methods in some important points related to empiricality, distribution shape of the population, sample size and repeated data. Parametric methods are used in the conclusions of the population based on empirical distribution. However, if there is not enough information about the population, some problems may be encountered in this transition. In this point, parameter estimation can be done with the help of resampling methods based on emprical distribution. Classical statistical methods require many assumptions about the distribution shape of the population and require working with large samples. When the sample size is not sufficient and the parametric assumptions do not occur, classical methods may be inadequate for the solution. Working with small samples is one of the most important advantages of resampling. In this case, the use of less presumptive resampling methods can be considered as an advantage. In addition, resampling methods include highly successful methods if the observed data is repeated.

METHODS
Resampling methods, such as Bootstrap and Jackknife, are used to obtain an estimate of the standard error of interest, confidence interval and distribution. In the Bootstrap method introduced by Bradley Efron (1979), new samples are created with the same size as the observed dataset and the estimation is made based on these new samples. After than a statistic is obtained for each of the new samples that are created, and the empirical distribution of this statistic is used to estimate the population parameter. In Jackknife method developed by Maurice Quenouille

(1949) and developed by John W. Tukey, new sample is obtained by excluding one sample value at a time. For this reason it is also known as delete-one jackknife technique and is appropriate to apply the data sets with wide range and extreme observations

Emprical Distribution Function and The Bootstrap Estimation of the Standart Error

In most of the statistical problems, the estimation is done with the help of the distribution which is taken from the sample. If the shape of the distribution is unknown, the theoretical information developed to create the distribution is used or the methods that do not require assumption about the shape of the distribution are consulted. The bootstrap method is a good helper when researchers do not have any information about sampling distribution. It attitudes to the sample like a real population and performes the estimates by creating emprical distribution.

In order to explain the emprical distribution assume that we have taken a random sample $x_i = (x_1, x_2, \cdots, x_n)$ from the population with F distribution and that this is the original data set. Each value in the sample is a discrete random variable and the definition range is $(x_i, 1 \le i \le n)$.The emprical distribution function is the cumulative distribution function of this random variable defined as in Eq.1. and the probability of taking each value ofx_i is$1/n$.

$$\hat{F}(x) = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

The bootstrap method is a resampling method that estimates a population parameter with F distribution by means of this emprical distribution. By "n" times sampling from the original dataset sample, "B" independent bootstrap samples are created, each with the same size as the dataset. When the data set of the emprical distribution is expressed as $\hat{F} \to (x_1^*, x_2^*, \cdots, x_n^*) = x^*$, Bnumber of independent bootstrap samples can be expressed as $x^{*1}, x^{*2}, \cdots, x^{*B}$. The bootstrap samples generated from the original dataset are summarized in the Table.1:

Table 1. Bootstrap Samples Generated from the Original Example

1. Bootstrap sample	x^{*1}	$x_1^{*1}, x_2^{*1}, \cdots, x_n^{*1}$
2. Bootstrap sample	x^{*2}	$x_1^{*2}, x_2^{*2}, \cdots, x_n^{*2}$
3. Bootstrap sample	x^{*3}	$x_1^{*3}, x_2^{*3}, \cdots, x_n^{*3}$
....	....	....
b. Bootstrap sample	x^{*b}	$x_1^{*b}, x_2^{*b}, \cdots, x_n^{*b}$
....	....	....
B. Bootstrap sample	x^{*B}	$x_1^{*B}, x_2^{*B}, \cdots, x_n^{*B}$

Based on these generated samples, the bootstrap estimate of the standard error can be obtained. For this purpose the standard deviations for each bootstrap sample are created by using the distribution of the sample. The standard deviations obtained for sample B are expressed as $s(x^{*1}), s(x^{*2}), \cdots, s(x^{*B})$. Here, the standard deviation of the replicates of the bootstrap, in other words, the bootstrap estimate of the standard deviation is expressed by the following equation:

$$\widehat{se}_{boot} = \{\sum_{b=1}^{B}[s(x^{*b}) - s(\cdot)]^2/B = 1\}^{1/2} \quad (2)$$

In the Eq.2 $s(\cdot)$can be expressed as $s(\cdot) = \sum_{b=1}^{B} s(x^{*b})/B$. The number of replicates required to estimate the standard error is at least 200. However, this value reaches much larger numbers when calculating confidence intervals. For example, it is recommended that the number of replicate samples be at least 1000 in order to obtain more accurate results than the percentages estimates for confidence intervals. The success of test results based on the bootstrap method depends on the number of replicates used to create bootstrap samples (Davidson, R. And MacKinnon, J.G., 1998). Many studies by researchers support this result. The number of bootstrap B replicates plays an active role in standard error calculations and the main goal is to reduce the standard error and obtain effective estimates. For an ideal bootstrap it is clear that the number of the replicates must approach to infinite value$(B \to \infty)$. Considering this situation in practice $\widehat{se}_{\infty} = \widehat{se}_{\hat{F}}(\bar{x})$ is obtained and this value is shown as in Eq. 3.

$$\widehat{se}_{\hat{F}}(\bar{x}) = \sigma_{\hat{F}}/\sqrt{n} = (\sum_{i=1}^{n}(x_i - \bar{x})^2/n^2)^{1/2} \quad (3)$$

The standard error takes the smallest value for $n \to \infty$. This will be the ideal bootstrap estimate of the standard error.

Bootstrap-t Confidence Interval

In the case of the bootstrap method used to create the standard normal ranges when the normal distribution assumption is not satisfied, the "z" bootstrap values for each of the Bsamples are calculated. The bootstrap table has a percentage of these B values. When it is assumed that the population parameter of interest is θ and $\hat{\theta}$is the estimate of this parameter the z values are calculated as in Eq.4.

$$z^*(b) = \frac{\hat{\theta}^*(b) - \hat{\theta}}{\widehat{se}^*(b)} \quad (4)$$

In this equation,$\hat{\theta}^*(b) = s(x^{*b})$ is the value of$\hat{\theta}$ for the bootstrap sample x^{*b} and $\widehat{se}^*(b)$ is the estimation of the standart error for bootstrap sample x^{*b}(Buur, D., 1994). α. percentage value of the $z^*(b)$can be obtained by means of $\hat{t}^{(\alpha)}$ as in $\{z^*(b) \le \hat{t}^{(\alpha)}\}/B = \alpha$(Efron, B., Tibshirani, R.J. , 1993). As a result the confidence interval for the θcan be expressed $(\hat{\theta} - \hat{t}^{(1-\alpha)}\widehat{se}, \hat{\theta} - \hat{t}^{(1-\alpha)}\widehat{se})$. The lower and upper limits are respectively expressed $\hat{\theta}_{lower} = \hat{\theta}^{*(\alpha)} = \hat{\theta} - z^{(1-\alpha)}\widehat{se}$ and $\hat{\theta}_{upper} = \hat{\theta}^{*(1-\alpha)} = \hat{\theta} - z^{(\alpha)}\widehat{se}$ and these values are 100α. and $100(1-\alpha)$. values of $\hat{\theta}^*$respectively. According to the researches made, the bootstrap-t confidence intervals obtained for large samples were found to approach the ranges obtained using standard normal or student-t distributions. It can also be stated that the bootstrap-t confidence interval method gives easier and more precise results than the other interval calculations (Chernick, M. R., 1999).

Bootstrap Percentage Confidence Interval

According to the emprical distribution $\hat{F}$ the bootstrap replicates are obtained for the original data set x^* and $\hat{\theta}^* = s(x^*)$are calculated for the distribution. After then the cumulative distribution function is created named as $\hat{G}$and the percentage confidence interval can be described as the α. and $(1-\alpha)$. values of the $\hat{G}$respectively in this distribution. These values correspond to $\left[\hat{\theta}_{\%lower}, \hat{\theta}_{\%upper}\right] = \left[\hat{G}^{-1}(\alpha), \hat{G}^{-1}(1-\alpha)\right]$. Because of the (100α). percentage of the distribution is $\hat{\theta}^*$, $\hat{G}^{-1}(\alpha) = \hat{\theta}^*$ is obtained and the confidence interval of the bootstrap percentage can be described as$\left[\hat{\theta}_{\%lower}, \hat{\theta}_{\%upper}\right] = \left[\hat{\theta}^{*(\alpha)}, \hat{\theta}^{*(1-\alpha)}\right]$. As the shape of the bootstrap distribution of the $\hat{\theta}^*$approaches normal, the standard normal and percent confidence intervals will be approximately equal. If $n \to \infty$increases according to the central limit theorem, the bootstrap histogram approaches normal but for small samples the shape of the distribution is away from the normal distribution. In this case, the standard normal and percent intervals will be different from each other. At this point, a number of criteria must be taken into account to decide which is more reliable. Before describing the criterion, it is useful to note that a non-parametric case of small sample size is addressed. The first criteria is to determine which distribution is closer to normal by looking at the shapes of the normal and percentage graphs for the sample covered. The distribution closest to the normal will give the appropriate confidence interval. The second criterion to be evaluated is the approximation of the intervals each other by a suitable logarithmic transformation to be performed on the θ. If the percent range and the normal range appear close to each other at the end of the transformation, the percentage interval is preferred, considering that transformation to the percentage intervals is easier. In addition, studies conducted so far have shown that percentage intervals give better and more accurate confidence intervals than normal intervals. It is meant by the good and correct confidence interval there; The bootstrap confidence interval is the ability to give accurate results in special cases.

Bias Corrected and Accelerated Confidence Interval

The bias-corrected and accelerated-confidence interval (BCA) is more advanced in terms of both the theory and practice of percentage confidence interval. According to the method the(100α).percentage of the $\hat{\theta}_1^*, \hat{\theta}_2^*, \cdots \hat{\theta}_B^*$ is $\hat{\theta}^{*(\alpha)}$ and the $(1-2\alpha)$. percentage confidence interval of itis$\left[\hat{\theta}_{\%lower}, \hat{\theta}_{\%upper}\right] = \left[\hat{\theta}^{*(\alpha)}, \hat{\theta}^{*(1-\alpha)}\right]$. The lower and upper limits of the BCa confidence interval are the percentage of the distribution (Efron,B., Tibshirani, R.J., 1993) and the limits are defined as $\left[\hat{\theta}_{\%lower}, \hat{\theta}_{\%upper}\right] = \left[\hat{\theta}^{*(\alpha_1)}, \hat{\theta}^{*(\alpha_2)}\right]$. α_1 and $\alpha_2$are given as follows:

$$\alpha_1 = \varphi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1-\hat{a}\left(\hat{z}_0 + z^{(\alpha)}\right)}\right) \qquad (5)$$

$$\alpha_2 = \varphi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1-\hat{a}\left(\hat{z}_0 + z^{(1-\alpha)}\right)}\right) \quad (6)$$

The bias correction factor $\hat{□}_0$found in Eq. 5 and Eq. 6 is found by the ratio of the number of values less than the original estimate $\hat{□}$to the number of repetitions of the bootstrap, and is expressed as(Efron, B.,).

$$\hat{□}_0 = □^{-1}\left(\frac{\#\hat{□}^*(□) < \hat{□}}{□}\right) \quad (7)$$

In the Eq.7, $□^{-1}$ is inverse of the standard normal cumulative distribution function. If the $\hat{□}_0$is equal to zero, exactly half of the $\hat{□}^*(□)$values are equal to or less than the$\hat{□}$. $\hat{□}$ is an acceleration factor and there are several ways to calculate this multiplier. For the estimation of the population parameter □, $\hat{□}$ acceleration factor can be defined as in Eq. 8.

$$\hat{a} = \frac{\sum_{i=1}^{n}\left(\hat{\theta}_{(.)} - \hat{\theta}_{(i)}\right)^3}{6\left\{\sum_{i=1}^{n}\left(\hat{\theta}_{(.)} - \hat{\theta}_{(i)}\right)^2\right\}^{3/2}} \quad (8)$$

In the Eq. 8, $\hat{\theta}_{(i)} = s\left(x_{(i)}\right)$ and$\hat{\theta}_{(.)} = \sum_{i=1}^{n}\hat{\theta}_{(i)}/n$.

Graphics-based testing methods are most often used to control the regression model in the resampling techniques. Especially it is used to test whether the residuals in the regression model are normal or approximate to normal. The standard normal percentage graph based on the bootstrap method can be used to test the approximate normal distribution of the distribution of the "□" bootstrap samples. The standard normal percentile graph can be thought of as a measure of the closeness of normality of replications. As the number of replications generated increases, graph approximates normal.

Jackknife After Bootstrap

In order to explain the jackknife method, it is necessary to be informed about the distribution.Each time an observation is excluded from the process according to the jackknife and the data set can be defined as $□_□ = (□_1, □_1, \cdots, □_{□-1}, □_{□+1}, \cdots, □_□)$. In the method□.jackknife sample is obtained by removing □. observation from the original data set and the □.observation value is calculated as $□_□ = □\bar{□} - (□ - 1)\bar{□}_□$(Walsh, B., 2000). The jackknife estimate of the standard error is given by the Eq. 9:

$$\widehat{□□}_{□□□□} = \left(\frac{□-1}{□}\sum_{□=1}^{□}\left(\hat{□}_□ - \hat{□}_{(.)}\right)\left(\hat{□}_□ - \hat{□}_{(.)}\right)\right)^{1/2} \quad (9)$$

In thisequation$\hat{\theta}_{(.)} = \sum_{i=1}^{n} \hat{\theta}_{(i)} / n$.

Standard error estimates for two different resampling methods have been obtained and researches have been done to determine which estimate is better than until today. The use of the jackknife estimation is recommended when the number of observations is low, while the bootstrap method provides advantages due to the large number of resampling replications. In this point, it can be said that the bootstrap method is more effective than the jackknife method and that the jackknife method is a method applied for control purposes after bootstrap method. In another respect, there is no distribution in the jackknife method when creating a bootstrap distribution with resamples based on the bootstrap method. To reveal the variability in the bootstrap estimates, it is known that the jackknife method is applied to the replications of bootstrap, which is known as jackknife after bootstrap. For this purpose, the bootstrap estimate of the standard error $\widehat{\square\square}_{\square\square\square\square(\square)}$is handled by removing an observation every time for the replications of the bootstrap. By applying the jackknife method to this obtained standard error, the jackknife estimate of the standard error is obtained. This estimation is named as jackknife after bootstrap estimation and the equation is given below:

$$\widehat{\square a \square}_{\square\square\square\square}(\widehat{\square\square}_{\square\square\square\square}) = [(\square - 1)/\square] \sum_{\square=1}^{\square} \left(\widehat{\square\square}_{\square\square\square\square(\square)} - \widehat{\square\square}_{\square\square\square\square(.)}\right)^2 \quad (10)$$

In the Eq.10, $\widehat{\square\square}_{\square\square\square\square(\square)}$ is defined as $\widehat{\square\square}_{\square\square\square\square(\square)} = \sum_{\square=1}^{\square} \widehat{\square\square}_{\square\square\square\square(\square)} / \square$.

Automotive Industry Sample

The aim of the study is to determine the most suitable production for the 1.6 and 1.8 injection vehicles produced by various automotive firms. Gasoline-powered, four-cylinder, hachback / sedan vehicles with common features were researched and compared over the years in the framework of the bootstrap method. The production information is evaluated in accordance with the standards and the normal quantile graphs drawn. Distribution charts for replications of bootstrap were tried to be determined as the closest to normal and the results were evaluated statistically. The relative advantages and disadvantages of bootstrap based confidence intervals are evaluated together. During the analysis, the vehicles were evaluated taking into account common features including engine power, weight and fuel consumption variables.

A review of the 1.6 and 1.8 injection models of Audi, BMW, Fiat, Honda, Mercedes, Mitsubishi, Opel, Renault and Volkswagen vehicles obtained 999 bootstrap replications from the data relating to the original engine variable. The results were evaluated based on three basic variables from a large number of variables: engine power, empty weight and fuel consumption. Because the application takes up a lot of space, the results are given only for the engine power variable.

The results of the analysis were also analyzed from the values for the replications of the bootstrap. The $s(x)$ statistic for each bootstrap sample is calculated and used as the mean value of the statistics. The data were first divided into 1.6 and 1.8 injection vehicles, then analyzed in terms of engine power variable. Confidence intervals for 999 bootstrap replications for variables were obtained and analyzes were performed by applying jackknife after bootstrap.

RESULTS

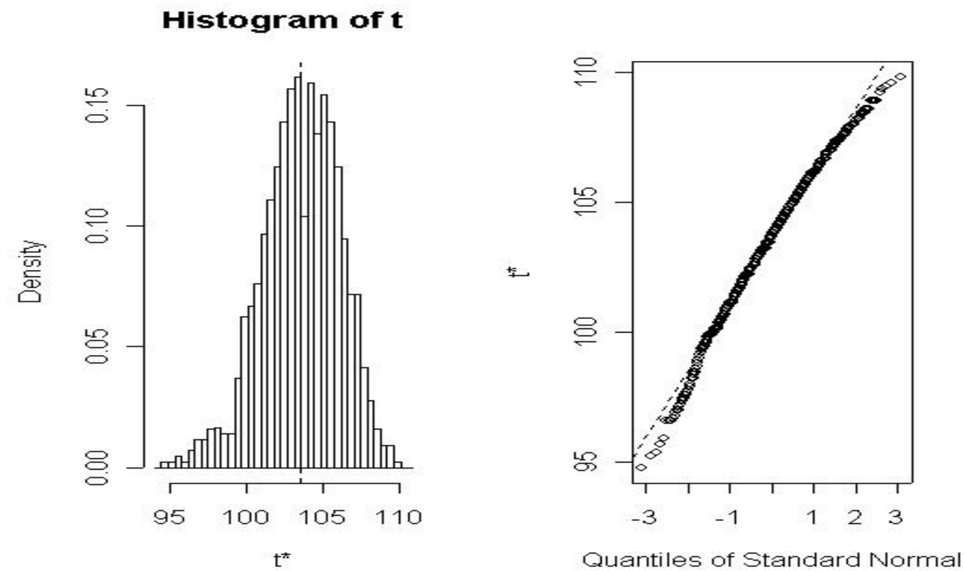
According to the analysis results obtained for 1.6 and 1.8 injection vehicles, it is seen that 1.6 injection vehicles are in conformity with the standards above 2003 in terms of engine power evaluation when assessed over all years for the variable. In 1.6 injection vehicles, 2003 bootstrap-t, bootstrap percentages and bootstrap bias were corrected and accelerated confidence intervals were found as in Table 2.

Table 2. Bootstrap-t, Bootstrap Percentage ve BCa Confidence Intervals for 1.6 Injection Vehicles for the Engine Power at Different Level

Level	Bootstrap-t	Percentages	BCa
%60	101.3, 105.7	101.4, 105.8	101.1, 105.4
%85	100.0, 107.2	99.9, 107.1	99.3, 106.7
%90	997., 107.8	99.3, 107.4	98.0, 107.0
%95	99.1, 109.2	97.9, 108.0	96.8, 107.4
%99	97.9, 111.2	95.9, 109.2	94.8, 108.3

A multidimensional comparison was made with regard to the evaluation of confidence intervals. The first comparison was made for the standard normal and the percent ranges. These two intervals approach each other as the graph of bootstrap replications approaches normal. When examined in terms of standard norms and percentages, this information is supported in 2003. Also, in 2003, the repetition of bootstrap was the closest to normal. On the other hand, the percentages should be preferred when the standard normal and percent confidence intervals are close to each other. In addition to this, bias corrected and accelerated confidence intervals offer more healthy results than percentages. The reason is that these intervals do not need parameter transformation and that non-parametric applications are preferred in most cases. It can be said that the range is the one that gives the best results within the confidence intervals based on the bootstrap method.

The bootstrap replications and the normal percentage (normal) quantile plots for the 1.6 engine power variable as follows for the year 2003:



(a) **(b)**

Figure 1 (a). 1.6i Distribution Graph for the Engine Power Variable
Figure 1 (b). 1.6i Normal Quantile Graph for the Engine Power Variable

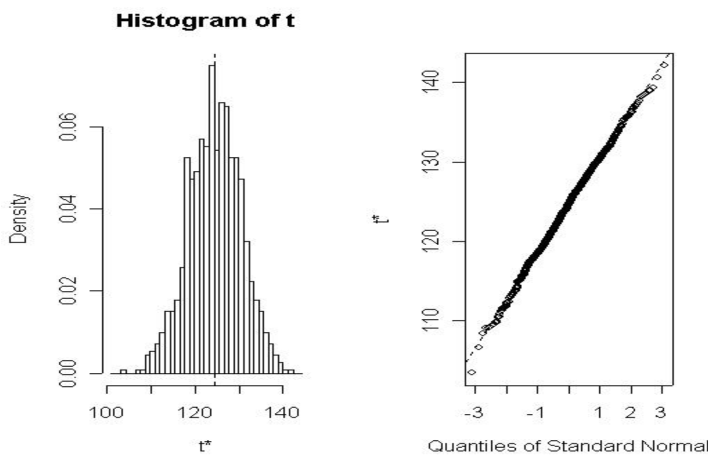
When the graph is examined, the year 2003 is determined as the year in which both the bias and the standard error are the lowest for the engine power variable. The bias was 0.005 and the standard error was 2.53 for 2003. The year 2003 when the bootstrap distribution graph is closest to normal is still the same.

In the case of 1.8 injection vehicles, the year 2002 shows the year in which the most suitable production is made to the standards for engine power variable. All the results for the 1.8 injection vehicles are as. in Table 3.

Tablo 3. Bootstrap-t, Bootstrap Percentage ve BCa Confidence Intervals for 1.8 Injection Vehicles for the Engine Power at Different Level

Level	Bootstrap-t	Percentages	BCa
%60	100.2, 108.9	128.4, 135.3	128.4, 135.3
%85	100.6, 109.4	125.8, 137.9	125.8, 137.9
%90	99.9., 107.1	125.0, 138.7	125.0, 138.9
%95	99.6, 109.2	123.8, 140.4	123.8, 140.6
%99	98.1, 114.9	121.4, 142.6	121.6, 142.7

The bootstrap replications and the normal percentage (normal) quantile plots for the 1.8 engine power variable as follows for the year 2002:



(a) **(b)**

Figure 2 (a). 1.8i Distribution Graph for the Engine Power Variable
Figure 2 (b). 1.8i Normal Quantile Graph for the Engine Power Variable

The jackknife after was applied over the original data set and the observations were deactivated one by one by creating bootstrap samples. It isexpectedthat the horizontal line in the middle part of the graph obtained by considering the sequencenumbersdoes not deviatetoomuch. The jab figure for the engine power change is as in Figure 3.

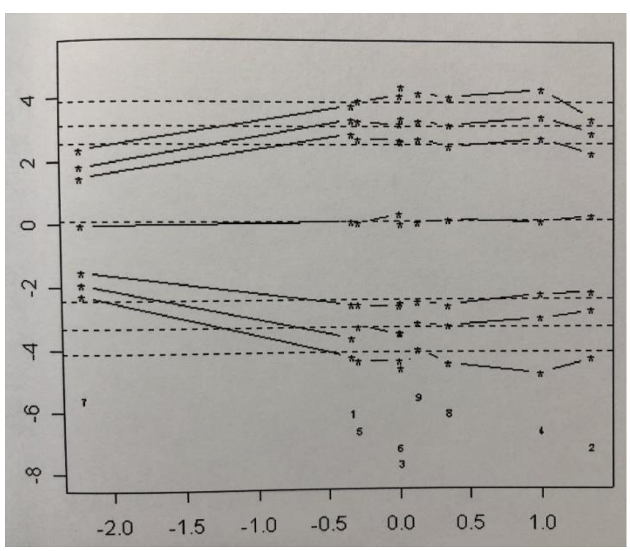


Figure 3. Jakknife After Bootstrap Plot

The x-axis in the graph indicates the jackknife values and the ordinate axis indicates the observed years. Zero on the ordinate axis corresponds to 2003 and it isalsoshown in this graph that 2003 was the best year in terms of engine power.

DİSCUSSİON

The meaningful results have been achieved in terms of the automotive sector with the help of numerous bootstrap replications obtained with fewer observations. When the results are evaluated in terms of the automotive sector, it is seen that 1.6 injection vehicles of automotive companies have met the standards in terms of engine power in 2003. As a result, it is seen that the BCA confidence interval determined is an expected range in terms of the automotive sector. In the event that the specified range is exceeded, that is, if the engine powers are excessively high or low, it is revealed that the manufacturer has sacrificed some of the characteristics of the vehicle. A high powered vehicle will consume more fuel, or an increase in fuel consumption will be seen provided that the empty weight of a vehicle in the lower engine power remains within the specified ranges.

The same assessment was made for 1.8 injection and it was observed that the year 2002 was the best year to standards and the confidence intervals determined in the mentioned year frame were at the desired level. For the companies that manufacture automobiles, the engine power, which is a decisive criterion in the plans to be made before production, will reduce the load that the consumer has to fold in terms of the car when staying within the mentioned ranges.

For example, it will consume less fuel. In today's conditions where fuel consumption is an important detail, the usefulness of the work done becomes more meaningful. While automotive companies plan to increase their engine power, they want the vehicle to have more features (comfort, safety etc) from one side, while the other side aims the vehicle to be at the same level in terms of fuel consumption with a lower class economic vehicle. While this is the case for automotive firms, consumers prefer vehicles that have the best features compared to their economic budgets. Firms that consider this demand of the consumers are also in competition with each other to offer the best means of consumers with different income levels. At this point, automotive firms try to bring the fuel consumption of vehicles as close to each other as possible, regardless of luxury or middle class vehicle. This also makes it clear that the confidence intervals for fuel consumption of 1.6 and 1.8 injected vehicles should be close to each other when considering the bootstrap confidence intervals for the purpose of the study. Techniques based on the bootstrap method, which is a statistically unprecedented work in the automotive sector, will lead the way to the next researcher.

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